

# Force Vectors

## Introduction

Everything from the paths around of Dublin to interstellar space is subject to different types of forces. Recognising such forces will allow us to better understand the interactions inside the nucleus of an atom, the construction of a bridge, or even the decay of a star into a supernova. The work of [Sir Isaac Newton](#) has played a vital role in advancing our knowledge of forces and his three laws underpin much of modern-day physics and mathematics. In this workshop, we will explore Newton's first law, which states that an object will remain at rest unless acted upon by an unbalanced force.

## Aim of Workshop

The aim of the workshop is to introduce students to force vectors in order to show them the applications of physics in the world around us. Students will also use their knowledge of algebra and trigonometry to examine the mathematical and physical significance of force vectors.

## Learning Outcomes

By the end of this workshop, students will be able to:

- Describe, in their own words, what is meant by a net force
- Draw unbalanced and balanced forces
- Solve problems related to force vectors

## KEY WORDS

### Force Vector

A quantity that describes both the magnitude and the direction of a force

### Net Force

The sum of all forces acting on an object

## Force Vectors: Workshop Outline

Suggested Time (Total mins)	Activity	Description
5 mins (00:05)	Introduction to Force	<ul style="list-style-type: none"> <li>Introduce the concept of a force and refer to Newton's second law (see <b>Appendix – Note 1</b>)</li> </ul>
15 mins (00:20)	Activity 1 Net force	<ul style="list-style-type: none"> <li>Explain what it meant by a force vector (see <b>Key Words</b>)</li> <li>Give examples of balanced and unbalanced forces (see <b>Appendix – Note 2</b>) and refer to the net force (see <b>Key Words</b>)</li> <li><b>Activity Sheet 1:</b> Students calculate the net force for the different boxes (see <b>Appendix – Note 3</b>)</li> </ul>
15 mins (00:35)	Activity 2 Vector Components	<ul style="list-style-type: none"> <li>Explain the horizontal and vertical component of a force vector and provide an example (see <b>Appendix – Note 4</b>)</li> <li><b>Activity Sheet 2:</b> Students attempt to find the vector components acting on the different boxes (see <b>Appendix – Note 5</b>)</li> </ul>
15 mins (00:50)	Activity 3 The Stubborn Donkey	<ul style="list-style-type: none"> <li><b>Activity Sheet 3:</b> Students calculate the force the donkey is exerting on the ropes (see <b>Appendix – Note 6</b>)</li> </ul>

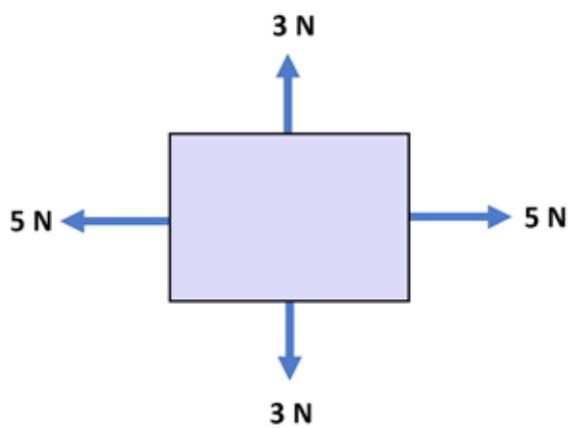
### Force Vectors: Workshop Appendix

#### Note 1: Force

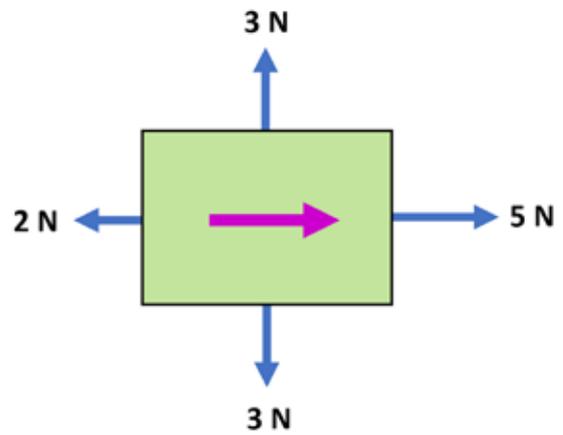
Force is any interaction that, when unopposed, causes a change in the motion of an object. This idea is summarised by Sir Isaac Newton in his Laws of Motion. For example, Newton’s second law describes force as the mass of an object times its acceleration, more commonly denoted by the formula,  $F=ma$ . The SI unit for force is Newton (N), with 1 Newton equal to  $1 \text{ kg} \times \text{m/s}^2$

#### Note 2: Balanced and Unbalanced Forces

The table below shows examples of balanced and unbalanced forces. Note: the pink arrow in each case represents the force vector.



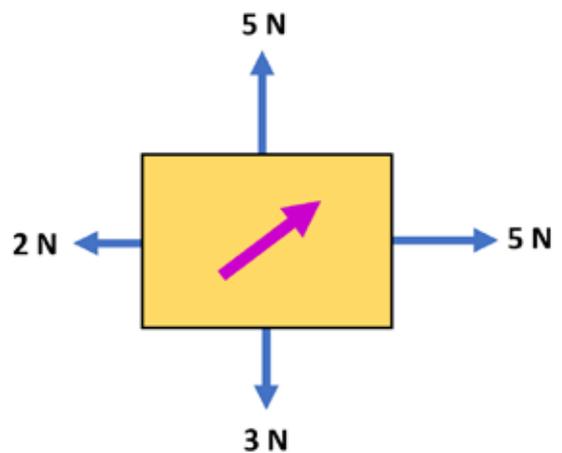
The forces above are said to be balanced as they ‘cancel’ each other out. There is thus a net force of 0 N and the box will remain stationary



The forces are unbalanced in the horizontal direction as there is a greater pulling force to the right. The net force is thus 3 N to the right



The forces are unbalanced, and the net force will be 7 N to the right since there is a pushing force of 4 N to the right in addition to a pulling force of 3 N to the right.



In this example, the forces are unbalanced in both the horizontal and vertical direction, and thus the box will move in a northeast direction.

**Note 3: Solutions for Activity 1**

Calculate the net force acting on each of the boxes below. Make sure to include the direction of the net force (i.e. left or right)

- |               |                |
|---------------|----------------|
| (a) 3 N right | (b) 5 N right  |
| (c) 0 N       | (d) 10 N right |
| (e) 6 N right | (f) 3 N right  |
| (g) 3 N left  | (h) 0 N        |

**Note 4: Vector Components**

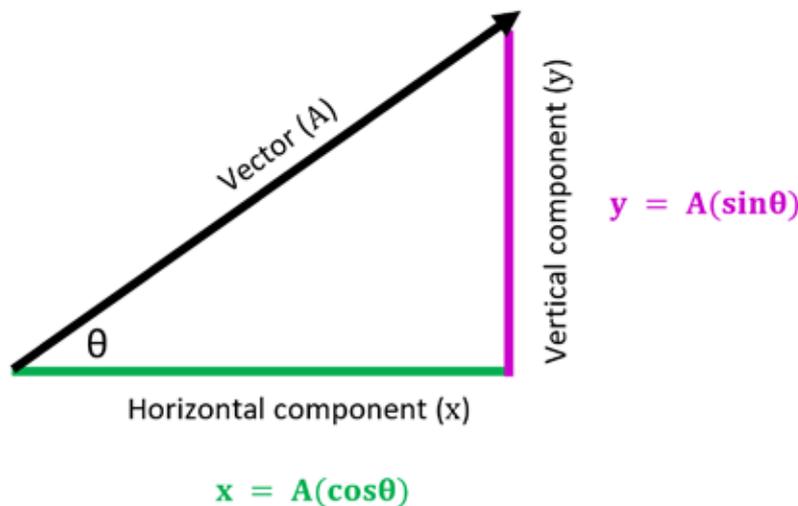
A vector acting on an object at an angle  $\theta$  can be broken down into its horizontal (x-axis) and vertical (y-axis) components as shown below. Since it is a right-angled triangle, we can find x and y using the trigonometric functions (Note: you may wish to revise this with the students).

$$\sin\theta = \frac{y}{A}$$

$$y = A(\sin\theta)$$

$$\cos\theta = \frac{x}{A}$$

$$x = A(\cos\theta)$$



**Example 1: Find the x and y-components acting on the box below**

Draw in the x and y-axes first and find the relevant angle.

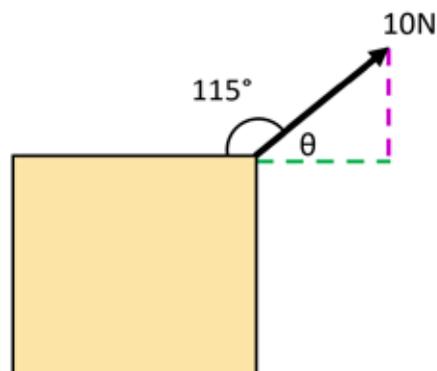
$$\text{Angle } \theta = 180^\circ - 115^\circ = 65^\circ$$

$$x = 10 (\cos 65^\circ)$$

$$x = 4.22 \text{ N}$$

$$y = 10(\sin 65^\circ)$$

$$y = 9.06 \text{ N}$$



**Example 2: Find the x and y-components acting on the box below**

Draw in the x and y-axes first and find the relevant angle.

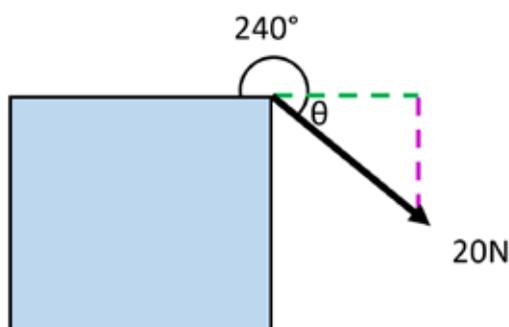
$$\text{Angle } \theta = 240^\circ - 180^\circ = 60^\circ$$

$$x = 20 (\cos 60^\circ)$$

$$x = 10 \text{ N}$$

$$y = -20 (\sin 60^\circ)$$

$$y = -17.32 \text{ N}$$



(y is negative since the vector is downward pointing. Consider the origin of the vector to be positioned at (0,0). We can thus identify the quadrants where x and y are positive and negative)

**Note 5: Solutions for Activity 2**

Find the vector components acting on the boxes below (Hint: Start by drawing the x and y-axes and think about which angle you need to use).

(a) x-component:  $20\sqrt{2}$  N

y-component:  $20\sqrt{2}$  N

(b) x-component: 24.62 N

y-component: 4.34 N

(c) x-component: 58.91 N

y-component: -27.47 N

(d) x-component: 49.24 N

y-component: -8.68 N

**Note 6: Solutions for Activity 3**

Q1. Two people are struggling to move their donkey as shown in the diagram below. Calculate the x and y-components for the force exerted by person A and by person B (i.e. the forces acting on the donkey).

**Person A:**

$$x = -80(\cos 75^\circ)$$

$$y = 80(\sin 75^\circ)$$

$$x = -20.71 \text{ N} \quad (\text{i.e. negative x-direction})$$

$$y = 77.27 \text{ N}$$

**Person B:**

$$x = 120(\cos 60^\circ)$$

$$y = 120(\sin 60^\circ)$$

$$x = 60 \text{ N}$$

$$y = 103.92 \text{ N}$$

**Q2. Now calculate the net force acting in the x and y-directions based on the forces exerted by person A and person B only.**

Person A is exerting a force of 77.27N in the y-direction and Person B is exerting a force of 103.92N in the y-direction. The net force is thus 181.19N in the positive y-direction.

Person A is exerting a force of -20.71N in the x-direction (see axes) and Person B is exerting a force of 60N in the x-direction. The net force is thus 39.29N in the positive x-direction.

**Q3. Hence calculate the x and y-components for the force the donkey must be exerting on the ropes to make the net force = 0 in both the x and y-direction.**

Let  $F_3 \sin(\theta)$  be the x-component and  $F_3 \cos(\theta)$  be the y-component for the force the donkey is exerting on the ropes

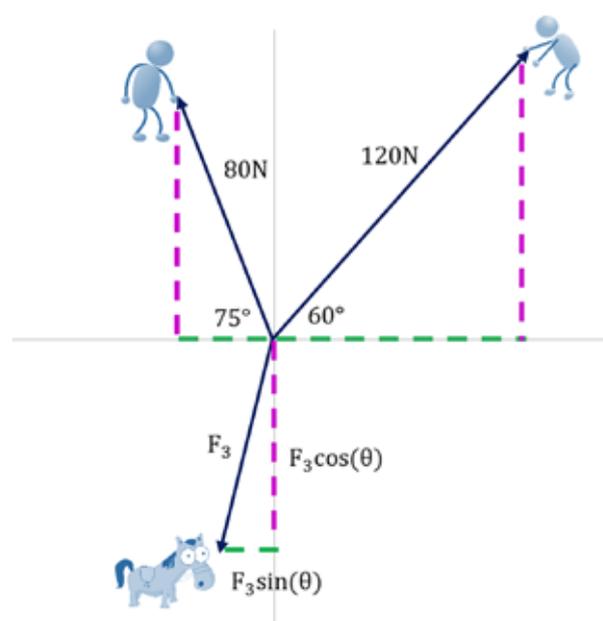
Since the people are unable to move their donkey, the net forces must equal 0 in both the x and y-directions. We thus have:

$$F_3 \cos(\theta) + 77.27\text{N} = 0$$

$$\Rightarrow F_3 \cos(\theta) = -77.27\text{N (y-component)}$$

$$F_3 \sin(\theta) + 39.29\text{N} = 0$$

$$\Rightarrow F_3 \sin(\theta) = -39.29\text{N (x-component)}$$



**Q4. Using Pythagoras' theorem, can you now find the force  $F_3$  that the donkey is exerting on the ropes?**

$$(F_3)^2 = (-77.27\text{N})^2 + (-39.29)^2$$

$$(F_3)^2 = 7514.357$$

$$F_3 = 86.69\text{N}$$

### Sources and Additional Resources

<https://physics.tutorvista.com/motion/force-vectors.html> (Vector forces)

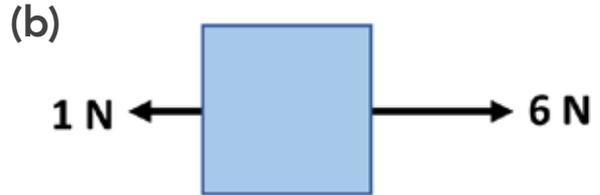
<https://www.physicsclassroom.com/class/vectors/Lesson-3/Resolution-of-Forces> (Vector components)

**Force Vectors: Activity 1**

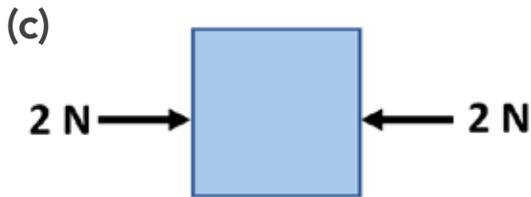
Calculate the net force acting on each of the boxes below. Make sure to include the direction of the net force (i.e. left or right)



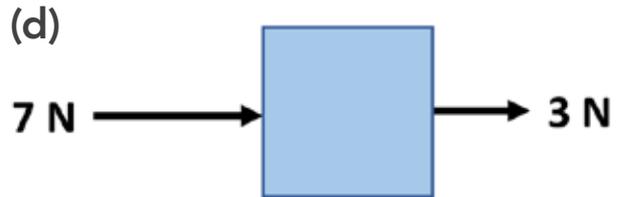
Net force: \_\_\_\_\_



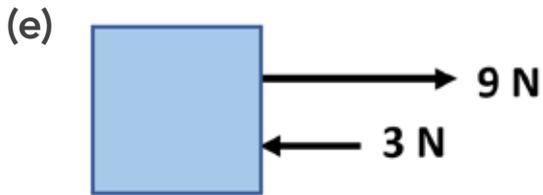
Net force: \_\_\_\_\_



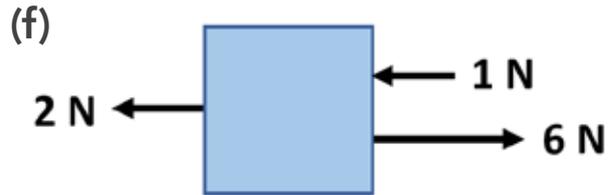
Net force: \_\_\_\_\_



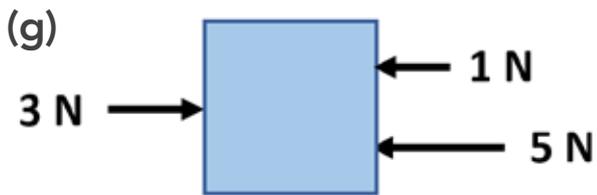
Net force: \_\_\_\_\_



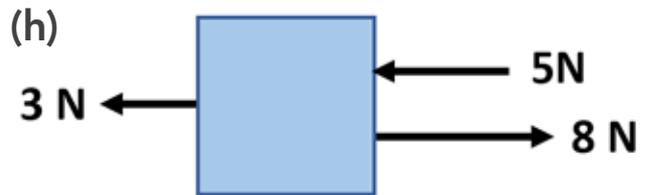
Net force: \_\_\_\_\_



Net force: \_\_\_\_\_



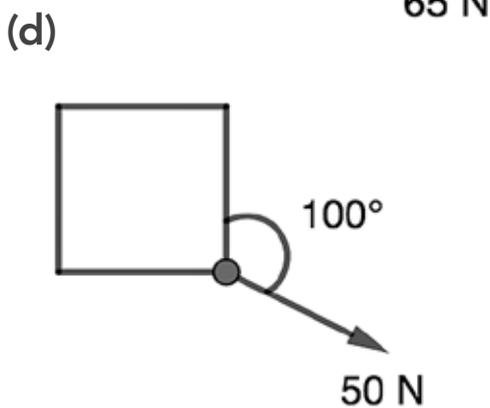
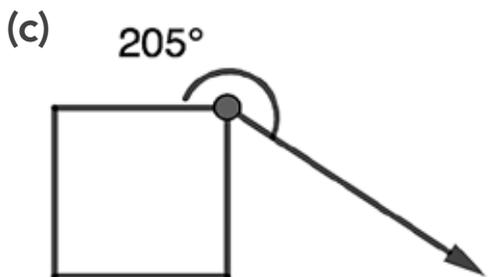
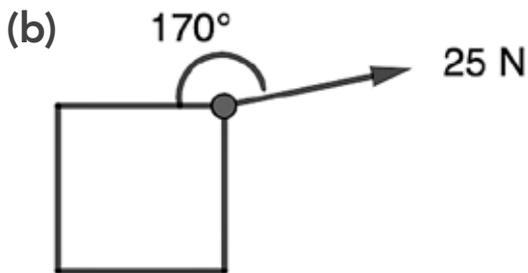
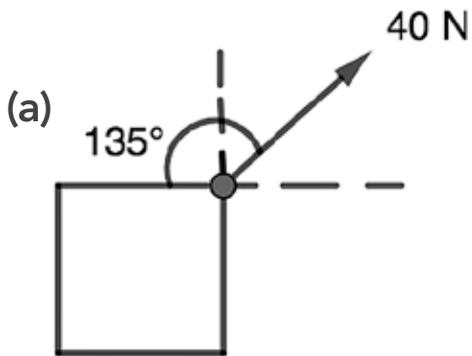
Net force: \_\_\_\_\_



Net force: \_\_\_\_\_

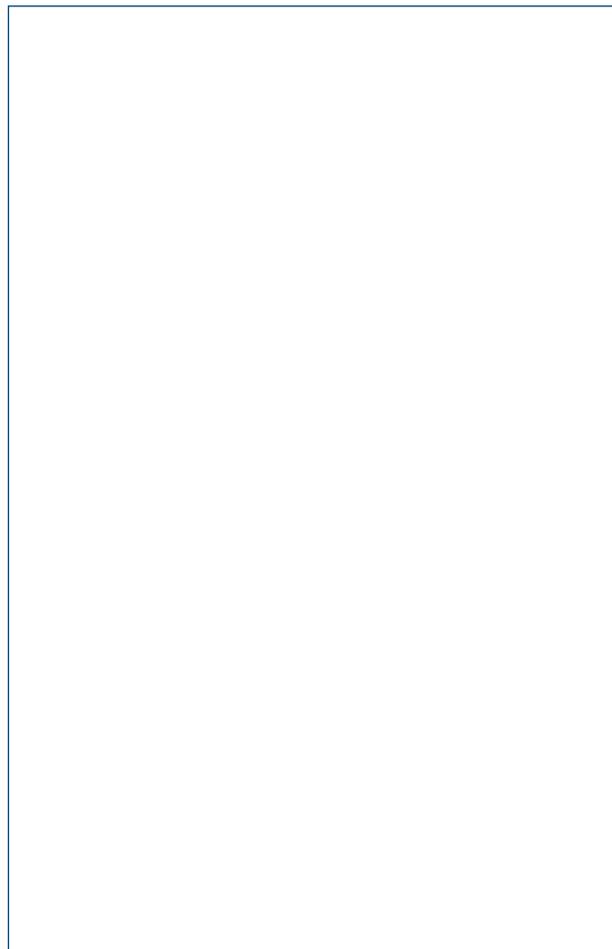
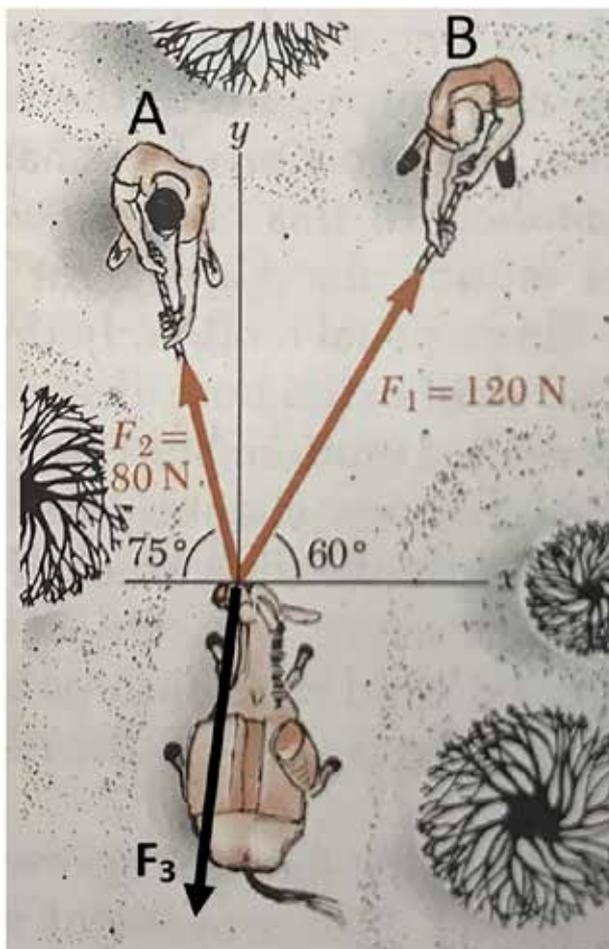
### Force Vectors: Activity 2

Find the vector components acting on the boxes below (Hint: Start by drawing the x and y-axes and think about which angle you need to use).

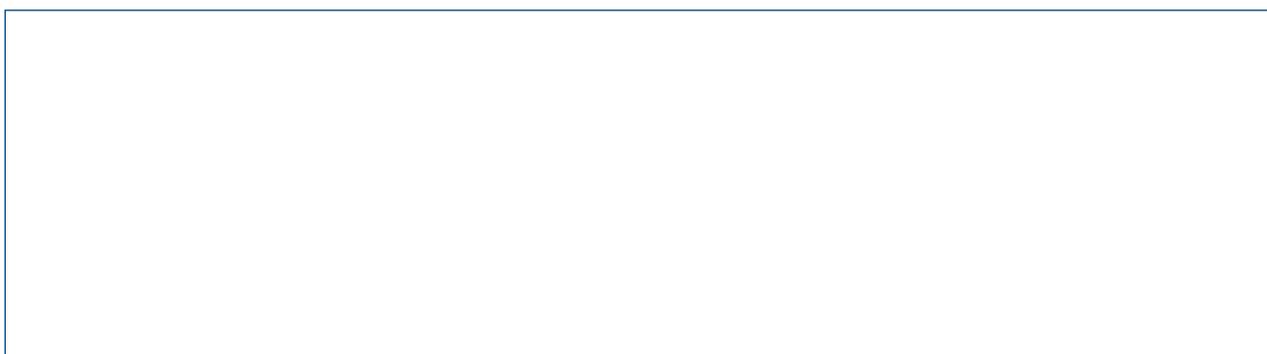


### Force Vectors: Activity 3

Q1. Two people are struggling to move their donkey as shown in the diagram below. Calculate the x and y-components for the force exerted by person A and by person B (i.e. the forces acting on the donkey). (Note: be careful with the signs – think about the x and y-axes).



Q2. Now calculate the net force acting in the x and y-directions based on the forces exerted by person A and person B only. You may wish to draw force vectors to help you.



Q3. Hence calculate the x and y-components for the force the donkey must be exerting on the ropes to make the net force = 0 in both the x and y-direction (Note: let  $F_3$  be the magnitude of the force the donkey is exerting on the ropes and  $\theta$  be the angle of the vector).

Q4. Using Pythagoras' theorem, can you now find the force ( $F_3$ ) that the donkey is exerting on the ropes?